

In-depth Insights into Utilization of Microsoft Excel Matrix Functions Technique for Adsorption Equilibrium Models

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Abstract—Adsorption equilibrium models are commonly utilised to analyse the mechanism of adsorption characteristics in a set system and establish the mass-transfer saturation mechanisms. Previous studies were mainly focused on the description and utilization of the adsorption equilibrium models, evaluation of the appropriate quality, estimation of the model's parameters and practical application of adsorption process in the environmental pollution control systems such as water and wastewater treatment. However, insights into utilization of Microsoft Excel Matrix Functions Technique for adsorption equilibrium models and their mathematical relations were rarely revealed in the literature. This paper provided step by step techniques required in the utilization of Microsoft Excel Matrix Functions in the determination of adsorption equilibrium parameters. Adsorption equilibrium data were obtained from our previous and published articles. Unknown engineering design parameters in the commonly utilized 2-parameters adsorption equilibrium models were determined using matrix functions and Microsoft Excel Solver in Microsoft Excel. Efficacies of these techniques were evaluated statistically using Analysis of Variance, Coefficient of Determination (CD) and correlation coefficient (R). and compared with experimental adsorption equilibrium. The study revealed that there was no significant difference between the adsorption capacities obtained using the two techniques at a 95 % confidence level. It was concluded that the CD and R between experimental adsorption equilibria, Microsoft Excel Solver and matrix simulated adsorption equilibria were between 0.6916 and 0.9535; 0.0492 and 0.9535; and 0.8318 and 0.9765, and 0.2218 and 0.9765, respectively. The paper established that matrix functions and Microsoft Excel Solver in Microsoft Excel are essential tools in the determination of engineering parameters in adsorption process.

Keywords— Adsorbent, Adsorption Isotherm, Arsenic, Matrix Technique, Microsoft Excel Solver.

I. INTRODUCTION

Environmental pollutions (water, air and land) are serious universal issue disturbing the environment and human health [1]. The cumulative number of various pollutants, which include conventional pollutants and emerging pollutants in the surface and groundwater bodies through industrial activities make water unsafe for human and animal consumption [2-7]. Improper management, removal and treatment of these contaminants in both surface and groundwater bodies originate water-related, water-borne and water-causing diseases. In addition, long-term exposure to these pollutants in the environment such as water bodies can gave room to a

variety of health issues, such as cancer, developmental organ disorders, and reproductive troubles [8-13]. There are several methods and techniques that have been utilised for the removal of these pollutants as part of water and wastewater treatment [14-28]. Figure 1 presents some of the treatment techniques for the removal of these pollutants from the environment. Figure 2 presents comparisons of these techniques for pollutants removal from water and wastewater. These techniques for pollutants include sedimentation, iron exchange, chemical precipitation, membrane filtration and adsorption [29-41]. From all these techniques adsorption has been described as one of the best methods and techniques for the removal of toxic substances through the use of adsorbent [4-16]. Adsorption process is a typical mass transfer process in which the pollutants (conventional or emerging) from the liquid phase move to the solid adsorbent [29-35]. Adsorption treatment process is one of the utmost used technologies in water and waste water treatment, based on its many advantages (easy maintenance, simple design, low price, and high efficiency[8-41]). Adsorption equilibrium models provide information and data on the adsorption rate, the mass transfer mechanisms and the performance of the adsorbent used, which indicates that accurate adsorption equilibrium parameters are essential tools for the design of the adsorption treatment systems. This study therefore evaluated the capabilities of Microsoft Excel Matrix Functions Technique for Adsorption Equilibrium Models in relation to experimental data and simulated equilibrium data from Microsoft Excel Solver.

II. LITERATURE REVIEW

Analysis of the adsorption equilibrium data can be conducted through linearized, non-linear, and least squares analysis using software such as Sigma Plot, Origin, Microsoft Excel Solver, MATLAB, Mathematica, Mathcad, Spreadsheet, Maple, GAUSS, Freemat, Flepro, Dataplot, Ardnial and SPSS. With reference to the cost of some of these software, there is a need to utilize tools and software that can be used to analyse data at no additional cost such as calc spread sheet on Libre Office, Microsoft Add-ins and Microsoft Excel. It has been documented that the functionality of Microsoft Excel can significantly increase the robustness of numerical data analysis when properly utilised and harnessed [42-46], but literature on utilization of matrix functions in Microsoft Excel is rare.

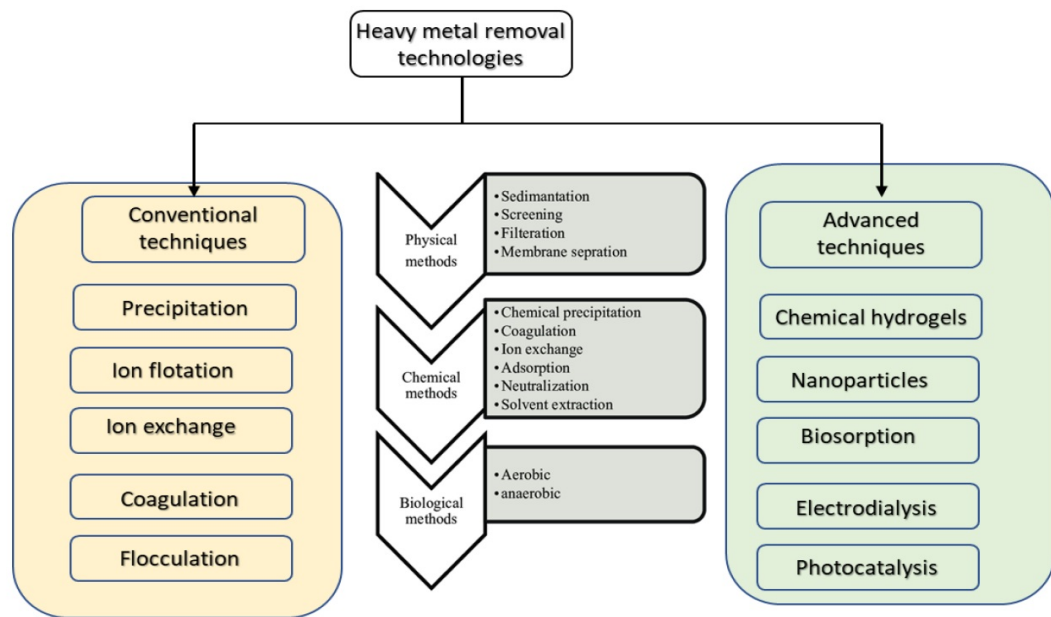


Fig. 1 some of the frequently used treatment techniques for the removal of pollutants from the environment (Source: Abiodun et al.[47] and Obijole et al.[48])

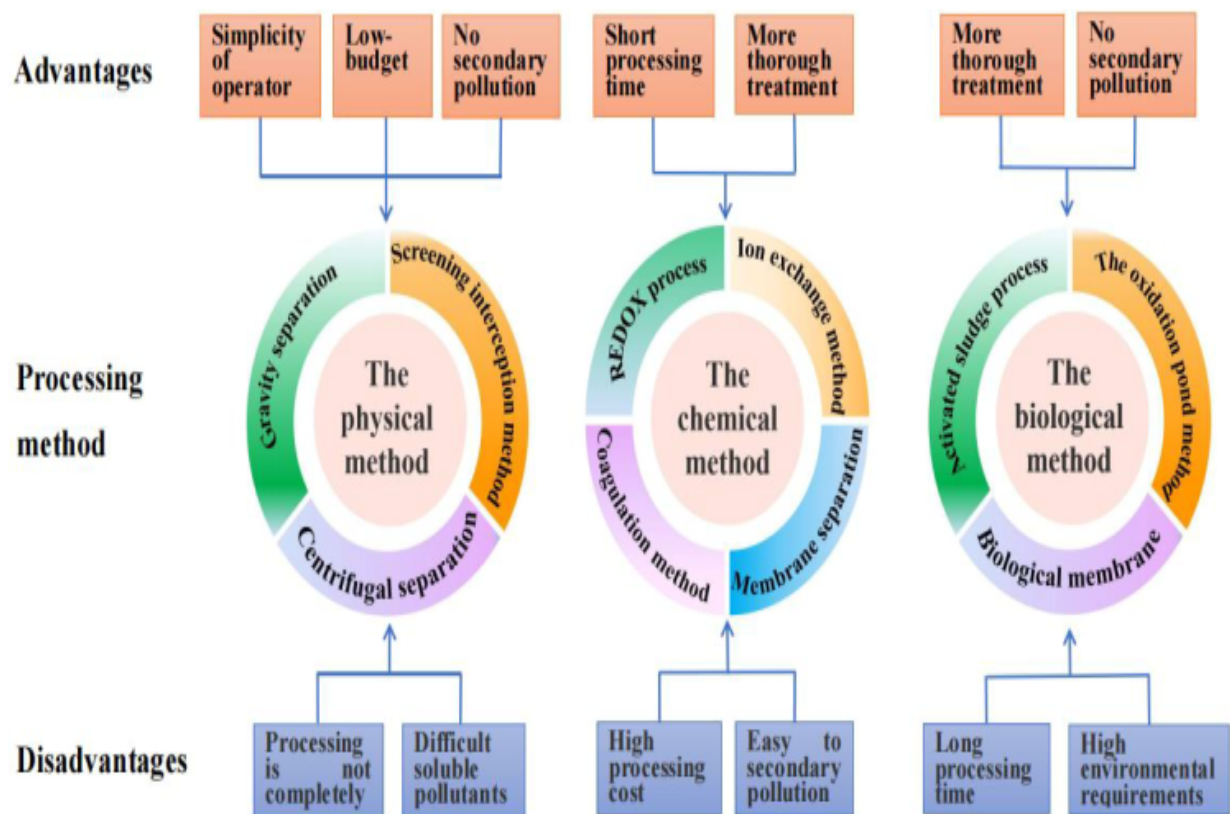


Fig. 2. Comparison of advantages and disadvantages of traditional water and wastewater treatment methods (Adopted from Sun *et al.*, 2022)

It is well stated that using matrices for solving numerical problems offers numerous advantages in handling complexity, improve computational efficiency, facilitate data analysis, and enhance the accuracy of models and predictions. This approach is particularly beneficial for multicomponent systems, dynamic processes, and optimization tasks, making it a powerful tool in both research and industrial applications of

adsorption technology. The main emphasis of this paper is to [42]:

- solve selected commonly used adsorption equilibrium models use matrix functions in Microsoft Excel (Table 1 presents in detail 2-parameters adsorption equilibrium) to determine the unknown parameter in the adsorption equilibrium models,

- b. establish the accurateness and preciseness of matrix transformation of the technique in solving commonly used adsorption equilibrium models with specific consideration to the utilization of least squares transformed and built-in matrix functions in Microsoft Excel, and
- c. compare the results with experimental adsorption equilibrium, and simulated adsorption equilibrium from Microsoft Excel Solver with a critical aim of attaining sustainable development goals 3 (good health and well-being), 6 (clean water and sanitation); 14 (life below water) and 15 (life on land) [49] and to further knowledge in computer education.

Matrix functions were selected based on accessibility, availability at no additional and easy to use.

III. METHODOLOGY

Data on arsenic removal from surface water such as Aponmu River, Ilara-Mokin, Nigeria by adsorption process using powdered eggshells and the adsorption properties of powdered eggshells were extracted from previous and published studies such as Amoko et al.[50]; Amoko et al. [51]; Fehintola et al.[52]; Obijole et al. [53] and Oke et al.[54]. Adsorption equilibrium models (Langmuir and Freundlich) were analysed using transformed linear and least squares form of the models. The Langmuir and Freundlich equations are as follows [42]:

$$q_e = \frac{a_L b_L C_e}{1 + a_L C_e} \quad (1)$$

Where;

q_e is the adsorption capacity of the absorbent at equilibrium(mg/g);

C_e is the final concentration of arsenic in the aqueous solution at equilibrium (mg/l);

α_L and β_L are the Langmuir adsorption constant(L/mg) and Langmuir isotherm parameter (mg/g) respectively

$$q_e = K_f C_e^{1/N_f} \quad (2)$$

Where;

K_f and α_f are the Freundlich isotherm exponents for pollutant mass per unit mass of the absorbent (g/mg) and Freundlich isotherm parameter for the volume of the

solution per unit mass of the pollutant (L/mg) respectively

The matrix arrays were solved with matrix functions in Microsoft Excel. The results were evaluated statistically using Analysis of Variance (ANOVA), Coefficient of Determination (CD) and correlation coefficient (R) and compared with experimental adsorption equilibrium and simulated equilibrium values from Microsoft Excel Solver. Figures 2 and 3 present the flowchart of the procedures for matrix and Microsoft Excel Solver techniques respectively. The mathematical and matrix transformation are as presented in equations (3) to (6). The details of the transformation are as presented in the appendix

$$Y = mX + C \quad (3)$$

$$\sum_{i=1}^n Y_i = m \sum_{i=1}^n X_i + nC \quad (4)$$

$$\sum_{i=1}^n X_i Y_i = m \sum_{i=1}^n X_i^2 + C \sum_{i=1}^n X_i \quad (5)$$

Matrix transformation of

$$\begin{pmatrix} \sum_{i=1}^n X_i & n \\ \sum_{i=1}^n X_i^2 & \sum_{i=1}^n X_i \end{pmatrix} \begin{pmatrix} m \\ C \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n Y_i \\ \sum_{i=1}^n X_i Y_i \end{pmatrix} \quad (6)$$

Table 1: Adsorption equilibrium isotherm models for single component aqueous solutions (Sources: Hamdaoui and Naffrechoux [55]; Rangabhashiyam *et al.*[56]; Vishali and Mullai [57]; Karri *et al.* [58]; Ayawei *et al.* [59]; Garba [60]; Al- Ghouti and Daana [61]; Outram *et al.*[62].

Model	Non-Linear Relationship	Linear Relationship	Parameters
Langmuir	$q_e = \frac{a_L b_L C_e}{1 + a_L C_e}$	$\left(\frac{C_e}{q_e}\right) = \frac{C_e}{b_L} + \frac{1}{b_L a_L}$ $\left(\frac{1}{q_e}\right) = \frac{1}{a_L b_L C_e} + \frac{1}{b_L}$ $q_e = b_L - \frac{b_L q_e}{a_L C_e}$ $\left(\frac{q_e}{C_e}\right) = b_L a_L - a_L q_e$ $\left(\frac{1}{C_e}\right) = b_L a_L \frac{1}{q_e} - a_L$	a_L and b_L
Freundlich	$q_e = K_f C_e^{1/N_f}$	$\text{Log}(q_e) = \text{log}(K_f) + \frac{1}{N_f} \text{Log}(C_e)$	K_f and N_f
Temkin	$q_e = \frac{RT}{b_t} \ln(a_t C_e)$	$q_e = a_t + 2.3 b_t \log C_e$	a_t and b_t
Dubinin–Radushkevich	$q_e = X_m \text{Exp}(-k\varepsilon^2)$	$\ln(q_e) = \ln(X_m) - k\varepsilon^2$	X_m and k
Halsey A	$q_e = \left(\frac{K_H}{C_e}\right)^{\frac{1}{N_H}}$	$\log(q_e) = \left(\frac{1}{N_H}\right) \log K_H - \frac{1}{N_H} \log C_e$	K_H and N_H
Halsey B (Hamdaoui and Naffrechoux, 2007)	$q_e = K_{Hf} \left(\frac{C_e}{C_0}\right)^{\frac{1}{N_{Hf}}}$	$\log(q_e) = \log K_H - \frac{1}{N_{Hf}} \log\left(\frac{C_e}{C_0}\right)$	K_{Hf} and N_{Hf}

Table 1: Continued

Model	Non-Linear Relationship	Linear Relationship	Parameters
Halsey C(Rangabhashiyum <i>et al.</i> , 2014)	$q_e = \exp\left(\frac{\ln(K_{Hr}) - \ln(C_e)}{N_{Hr}}\right)$	$\ln(q_e) = \frac{1}{N_{Hr}}(\ln(K_{Hr}) - \ln C_e)$	K_{Hr} and N_{Hr}
Frenkel- Halsey- Hill	$q_e = \exp\left(\frac{\ln((K_{fhh}) - \ln(C_e))}{N_{fhh}}\right)$	$\ln(q_e) = \left(\frac{1}{N_{fhh}}\right)\ln K_{fhh} - \frac{1}{N_{fhh}}\ln C_e$	K_{fhh} and N_{fhh}
Harkin Jura- A	$q_e = \left(\frac{(A_{hj})}{B_{hj} + \log(C_e)}\right)^{0.5}$	$\log(q_e)^2 = \log(A_{hj}) + \log(B_{hj} + \log C_e)$	A_{hj} and B_{hj}
Harkin Jura -B	$q_e = \left(\frac{(A_{hj})}{B_{hj} - \log(C_e)}\right)^{0.5}$	$\log(q_e)^2 = \log(A_{hj}) + \log(B_{hj} - \log C_e)$	A_{hj} and B_{hj}
Jovanovic	$q_e = q_m(1 - \exp^{-kC_e})$	$\log(q_e) = \log(q_m) + \log(1 - \exp^{-kC_e})$	q_m and k
Elovich	$\left(\frac{q_e}{q_{mE}}\right) = K_E C_e \left(\exp^{-\frac{q_e}{q_{mE}}}\right)$	$\ln\left(\frac{q_e}{C_e}\right) = \ln(K_E q_{mE}) - \frac{q_e}{q_{mE}}$	K_E and q_{mE}
Folwer –Guggenhien	$K_{FG} C_e = \frac{\theta}{1 - \theta} \exp\left(\frac{2\theta W}{RT}\right)$	$\ln\left(\frac{C_e(1 - \theta)}{\theta}\right) = -\ln(K_{FG}) + \frac{2\theta W}{RT}$	K_{FG} and W
Hill-de Boer	$K_{Hd} C_e = \frac{\theta}{(1 - \theta)} \exp\left(\frac{\theta}{(1 - \theta)} - \frac{K_2 \theta}{RT}\right)$	$\ln\left[\frac{C_e(1 - \theta)}{\theta}\right] - \frac{\theta}{(1 - \theta)} = -\ln(K_{Hd}) - \frac{K_2 \theta}{RT}$	K_{Hd} and K_2
Kiselev	$K_1 C_e = \frac{\theta}{(1 - \theta)(1 + K_n \theta)}$	$\frac{1}{C_e(1 - \theta)} = \frac{K_1}{\theta} + K_1 K_n$	K_1 and K_n
Flory - Huggins	$\frac{\theta}{C_e} = K_{FH} (1 - \theta)^{N_{FH}} \quad \theta = 1 - \frac{C_e}{C_0}$	$\log\left[\frac{\theta}{C_e}\right] = \log(K_{FH}) + N_{FH} \log(1 - \theta)$	K_{FH} and N_{FH}
Langmuir - Vageler	$q_e = \left(\frac{C_f q_{\max}}{C_f + K_{LV}}\right); \quad C_f = \frac{C_0 V}{m}$	$\log(q_e) = \log(C_f q_{\max}) - \log(C_f + K_{LV})$	$q_{\max}$ and K_{LV}

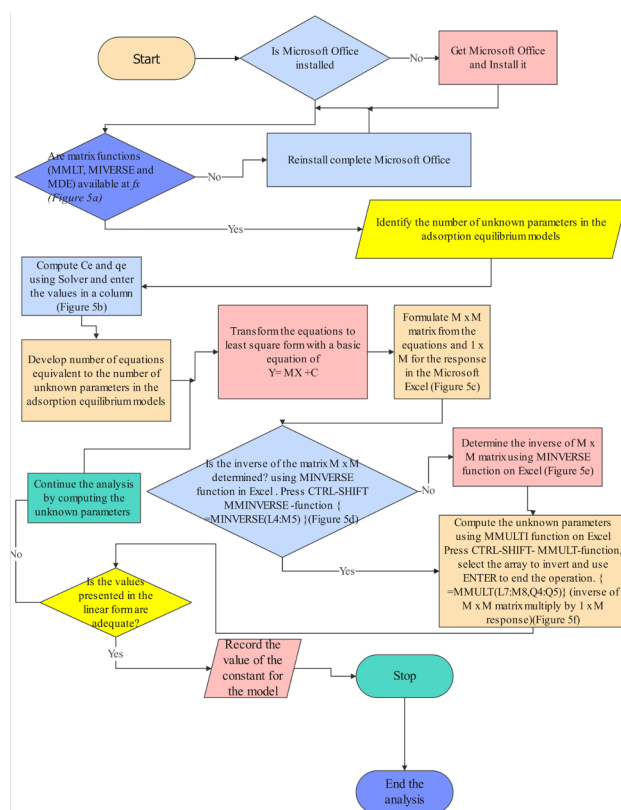


Fig. 2: Procedure for using the Matrix technique on Microsoft Excel in the computation of the Isotherm's parameters

IV. RESULTS AND DISCUSSION

Results from the study are presented as follows: properties of powdered eggshells, Step-by step of Microsoft Excel Matrix Functions techniques, Adsorption equilibrium model and parameters, efficacies evaluations of the technique.

a) Properties of powdered eggshells: The key properties of powdered eggshells in relation to adsorption are presented in literature such as Amoko et al.[50]; Amoko et al. [51]; Fehintola et al.[52]; Obijole et al. [53] and Oke et al.[54].

b) Step-by step of Microsoft Excel Matrix Functions techniques: Figures 4 and 5 provide data, information and deep insight into procedures needed to utilize Microsoft Excel Matrix functions and Microsoft Excel Solver to solve adsorption equilibrium equations. The figure provides the actions and solution to the flow chart provided in figures 2 and 3.

c) Adsorption equilibrium model and parameters: Table 2 presents the selected two adsorption equilibrium models with 2-parameters constants (Langmuir and Freundlich). The Table revealed that the values of adsorption equilibrium parameters vary with the adsorption equilibrium models, methods and techniques used (matrix and Microsoft Excel Solver), but the values were similar in magnitudes. This result indicates that the matrix technique in Microsoft Excel (in-built functions) can be used to solve numerical problems of 2-parameters adsorption equilibrium

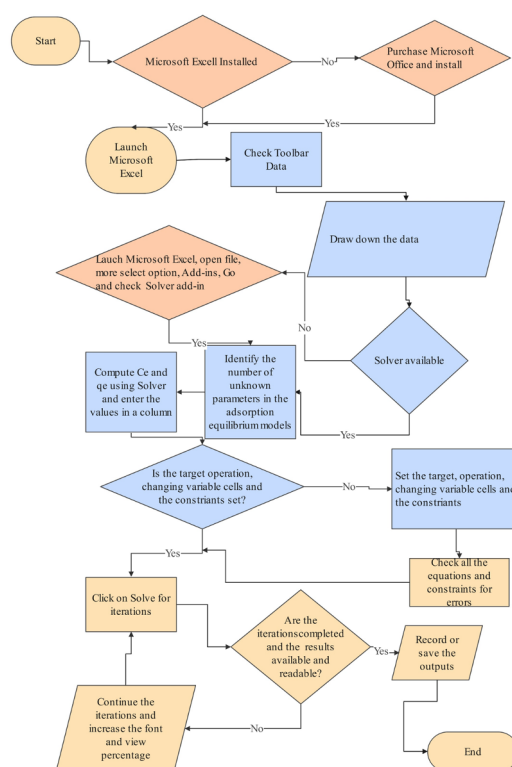


Fig. 3: Procedure for using Microsoft Excel Solver technique on Microsoft Excel in the computation of the Isotherm's parameters

models such as Langmuir and Freundlich. The Table also established that these 2-parameters adsorption equilibrium models must be linearized and transformed into matrix arrays for easier applications of Microsoft Excel Matrix functions.

- d) Table 3 shows the result of the effects of the technique of adsorption equilibria. The Table revealed that there was no significant difference between experimental adsorption equilibria and simulated adsorption equilibria (matrix and Microsoft Excel Solver) at a 95 % confidence level ($F_{1,22} = 0.175$, $p = 0.680$ which is greater than 0.05). for the matrix technique. Table 4 shows the result of the effects of the technique of adsorption equilibria parameters (adsorption capacities). There was no significant difference between the adsorption capacities obtained using the two techniques at a 95 % confidence level ($F_{12,39} = 0.772$; $p = 0.675$ which is greater than 0.05).
- e) **Efficacies evaluations of the technique:** Table 5 presents the results of the R for both matrix and Microsoft Excel Solver techniques. These values of R established that the relationship between experimental adsorption equilibria and simulated adsorption equilibria was between 0.2218 (Langmuir A) and 0.9765 (Langmuir D), which indicates that the relationship was between above average to good relationship as stated in literature such as Loveday[63]; Oke [64]; Oke et al.[65]; Suwamannahong et al.[66].

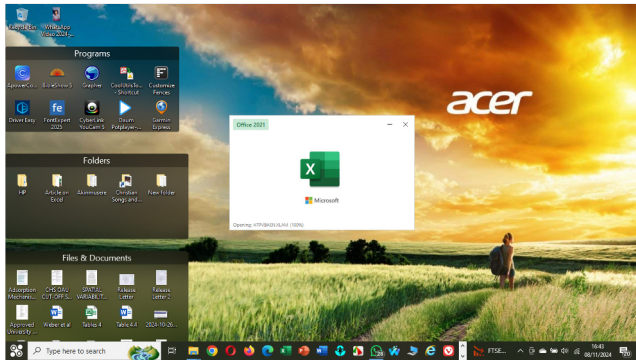


Fig. 4ai: Launching of Microsoft Excel on the desktop

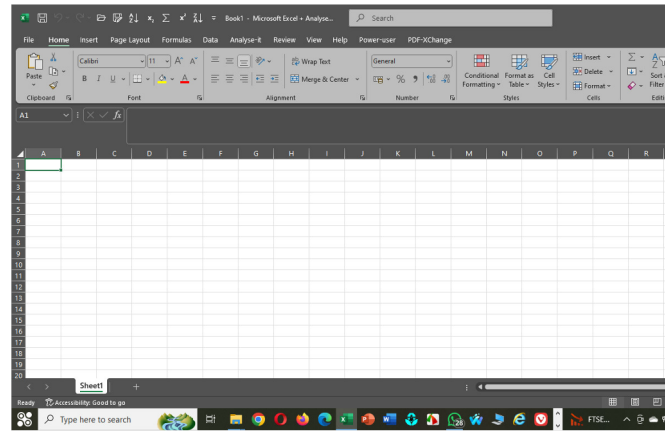


Fig. 4aii: Fully Launched Microsoft Excel

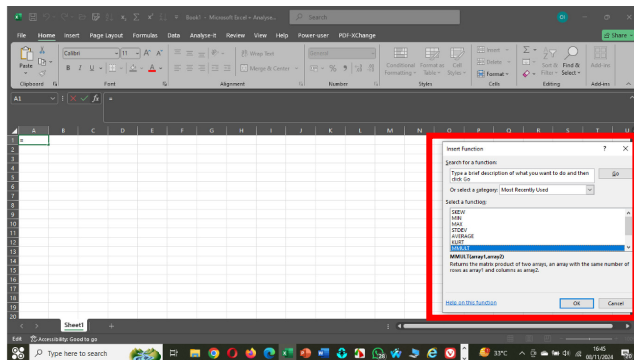


Fig. 4aiii: Fully Launched Microsoft with fx dialogue

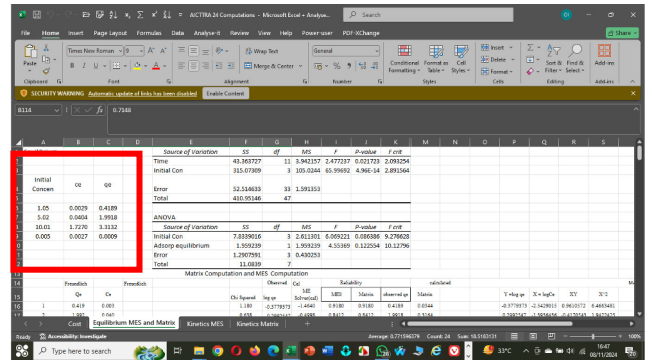


Fig. 4b: Fully launched Microsoft with Ce and qe entered

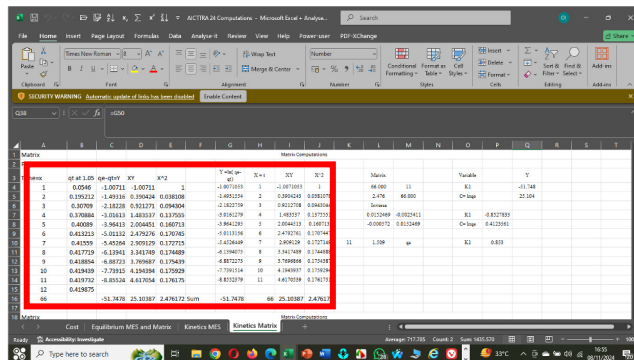


Fig. 4c: Fully launched Microsoft Excel with least square computations

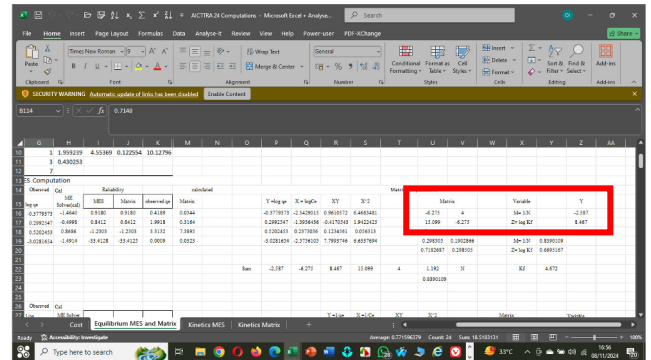


Fig. 4d: Fully launched Microsoft Excel with Matrix array entered

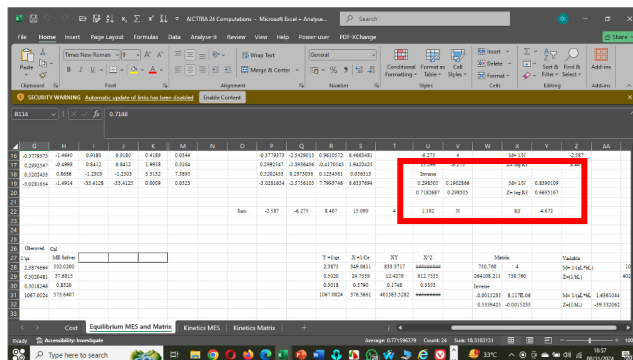


Fig. 4e: Fully launched Microsoft Excel with Matrix functions

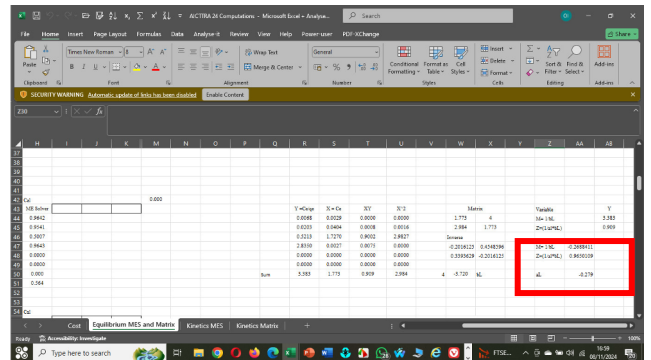


Fig. 4f: Fully launched Microsoft Excel with the final answers

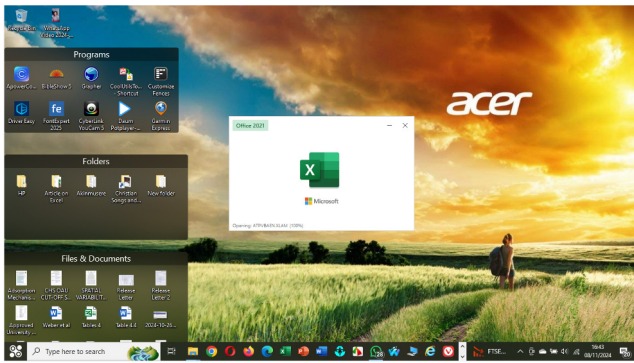


Fig. 5ai: Launching of Microsoft Excel on the desktop for MES

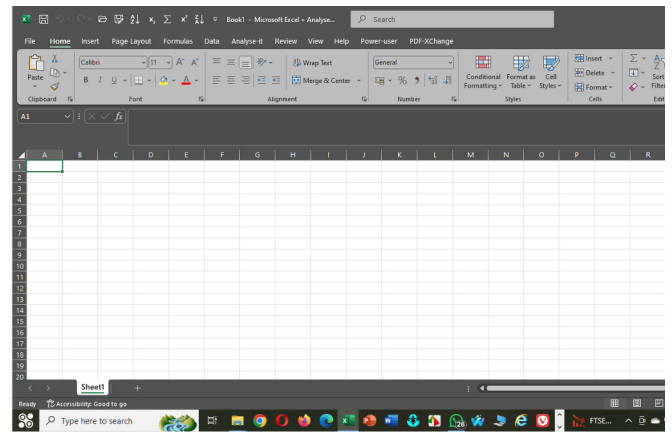


Fig. 5aii: Fully Launched Microsoft Excel for MES

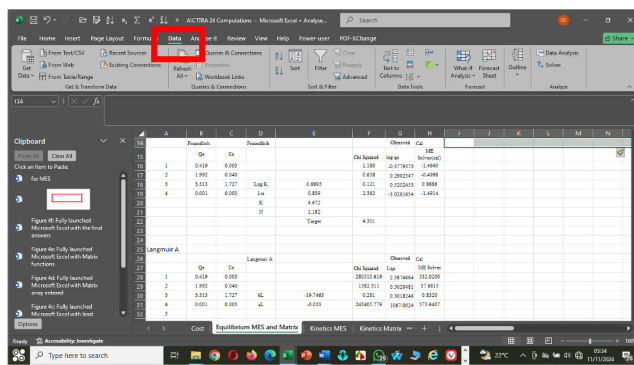


Fig. 5aiii: Fully Launched Microsoft with Data on toolbar MES dialogue

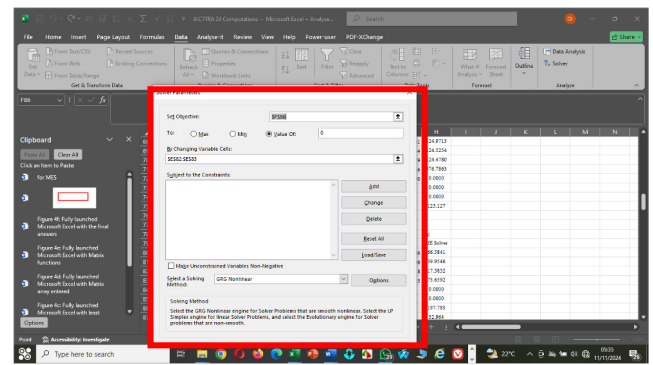


Fig. 5aiv: Fully Launched Microsoft with MES dialogue

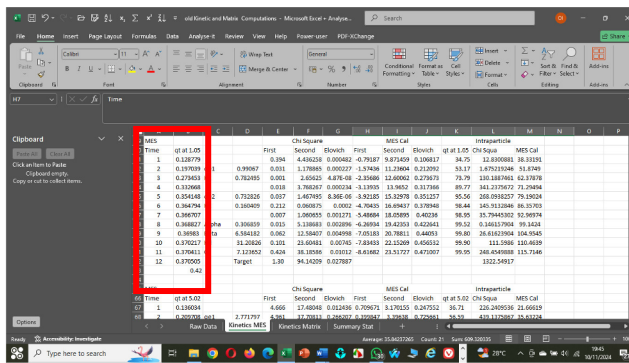


Fig. 5b: Fully launched Microsoft Excel with Ct and qt entered

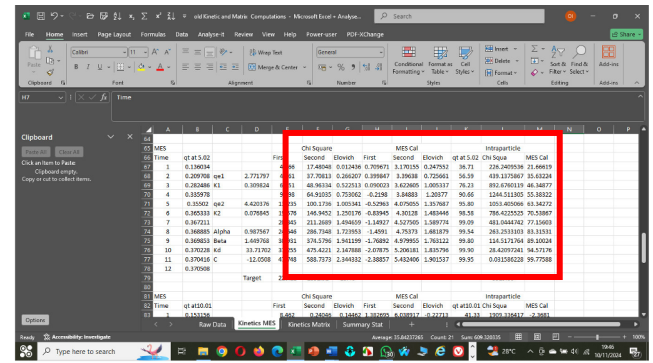


Fig. 5b: Fully launched Microsoft Excel with MES settings

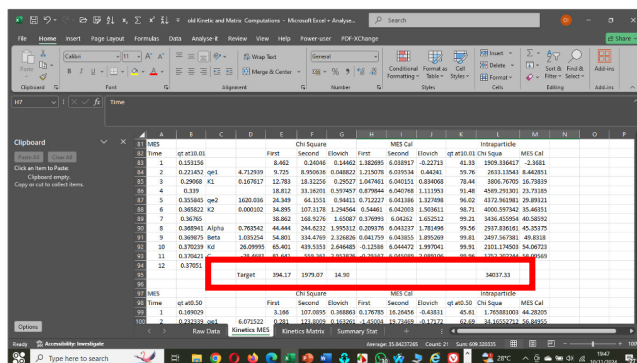


Fig. 5d: Fully launched Microsoft Excel with MES targets

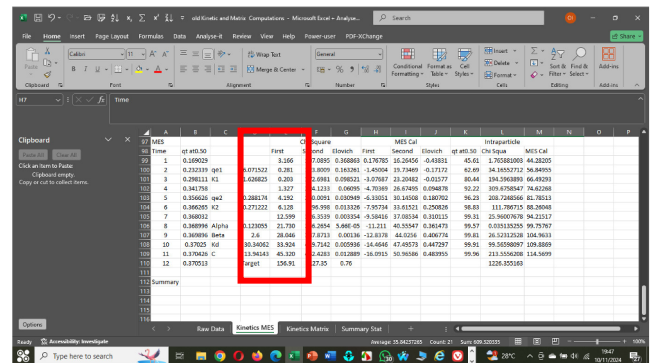


Fig. 5f: Fully launched Microsoft Excel with MES the final answers

Table 2 Adsorption equilibrium models with 2-parameters constants (Langmuir and Freundlich).[42]

Summary	Matrix	MES	Parameters
Freundlich	4.6721	4.6721	K
	1.1919	1.1919	N
Langmuir A	-0.0253	-19.7463	b _L
	-24.1623	-0.0332	a _L
Langmuir B	-3.7197	-3.7197	b _L
	-0.2786	-0.2786	a _L
Langmuir C	1.8444	1.4299	b _L
	-119.6585	-260.2039	a _L
Langmuir D	4.3519	4.3519	b _L
	16.9292	16.9293	a _L
Langmuir E	-0.0019	-0.0019	b _L
	-124.4066	-124.4066	a _L

Table 3 Result of the effects of the technique of adsorption equilibria [42]

Source of Variation	Sum of Squares	Degree of freedom	Mean Sum of square	F-value	P-value
Between the Constant from MES and Matrix Techniques	776.93274	1	776.9327	0.174967	0.679789
Within MES and Matrix Techniques	97689.764	22	4440.444		
Total	98466.697	23			

Table 4 Result of the effects of the technique of adsorption equilibria parameters (adsorption capacities)

Source of Variation	Sum of Squares	Degree of freedom	Mean Sum of square	F-value	P-value
Between the adsorption capacities from MES and Matrix Techniques	38.918271	12	3.243189	0.771571	0.674903
Within the adsorption capacities from MES and Matrix Techniques	163.93092	39	4.203357		
Total	202.84919	51			

Table 5: Results of the R for both matrix and Microsoft Excel Solver techniques

Statistical Evaluation	Freundlich	Langmuir A	Langmuir B	Langmuir C	Langmuir D	Langmuir E
Coefficient of Determination of Matrix Technique	0.7148	0.0492	0.6916	0.7748	0.9535	0.7798
Coefficient of Determination of Microsoft Excel Solver Technique	0.7148	0.6995	0.6916	0.8060	0.9535	0.7798
Correlation of Coefficient of Matrix Technique	0.8455	0.2218	0.8316	0.8802	0.9765	0.8831
Correlation of Coefficient of Microsoft Excel Solver Technique	0.8455	0.8364	0.8316	0.8978	0.9765	0.8831

V. CONCLUSION

The Based on the study and the findings, the following key conclusions can be drawn:

- Matrix technique with the Microsoft Excel function tools can be used to solve adsorption equilibrium equations.
- The performance of the technique is reliable and dependable as it predicted experimental data well.
- Using the matrix is flexible (the technique can take N x N matrix arrays of N greater than 100) and faster due to its capacities as in-built functions.

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Appendix A: Derivations Of Langmuir Linear Equations And General Form Of Least Square For Matrix Transformations

A. **Langmuir linear equation A Model (unknown parameters are a_L and b_L):** $q_e = \frac{a_L b_L C_e}{1 + a_L C_e}$ which indicates that

$$\frac{1}{q_e} = \frac{1 + a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{1}{b_L} \frac{1}{q_e} = \frac{1}{a_L b_L} \left(\frac{1}{C_e} \right) + \frac{1}{b_L}$$

where X stand for independent variables and Y stands for dependent variables

B. **Langmuir linear equation B Model:** $q_e = \frac{a_L b_L C_e}{1 + a_L C_e}$ which indicates that

$$\frac{1}{q_e} = \frac{1 + a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{1}{b_L} \text{ indicates that } \frac{C_e}{q_e} = \left(\frac{1}{b_L} \right) C_e + \frac{1}{a_L b_L}$$

C. **Langmuir linear equation C Model:** $q_e = \frac{a_L b_L C_e}{1 + a_L C_e}$ which indicates that

$$\frac{1}{q_e} = \frac{1 + a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{1}{b_L} ; b_L = \frac{b_L q_e}{a_L b_L C_e} + q_e \equiv q_e = b_L - \frac{q_e}{a_L C_e}$$

D. **Langmuir linear equation D Model:** $q_e = \frac{a_L b_L C_e}{1 + a_L C_e}$ which indicates that

$$\frac{1}{q_e} = \frac{1 + a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{1}{b_L} ; \frac{q_e}{C_e} = a_L b_L - (a_L) q_e$$

E. **Langmuir linear equation E Model:** $\frac{1}{q_e} = \frac{1 + a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{a_L C_e}{a_L b_L C_e} = \frac{1}{a_L b_L C_e} + \frac{1}{b_L}$

$$\frac{1}{C_e} = a_L b_L \left(\frac{1}{q_e} \right) - a_L$$

F. **Freundlich Model:** $q_e = K_f C_e^{1/N_f}$. Linear equation Freundlich Model

$$\text{Log}(q_e) = \log(K_f) + \frac{1}{N_f} \text{Log}(C_e)$$

APPENDIX B: Matrix Array Of Langmuir Linear Equations And General Form Of Least Square For Matrix Transformations

Table B1: Freundlich

Qe	Ce		Y =log qe	X = logCe	XY	X^2	Matrix		Variable		Y
0.419	0.003		-0.3779373	-2.5429015	0.9610572	6.4663481	-6.275	4	M= 1/N		-2.587
1.992	0.040		0.2992547	-1.3936436	-0.4170543	1.9422425	15.099	-6.275	Z= log Kf		8.467
3.313	1.727		0.5202453	0.2373036	0.1234561	0.056313	Inverse				
0.001	0.003		-3.0281654	-2.5756105	7.7993746	6.6337694	0.298505	0.1902866	M= 1/N	0.8390109	
							0.7182687	0.298505	Z= log Kf	0.6695167	
		Sum	-2.587	-6.275	8.467	15.099	1.192	N	Kf	4.672	

Table B 2: Langmuir A

Qe	Ce		Y =1/qe	X =1/Ce	XY	X^2	Matrix		Variable		Y
0.419	0.003		2.387	349.061	833.372	6.4663481	750.760	4	Variable		1070.194
1.992	0.040		0.502	24.754	12.428	1.9422425	264108.211	750.760	M= 1/(aL*bL)		402429.502
3.313	1.727		0.302	0.579	0.175	0.056313	Inverse		Z=(1/bL)		
0.001	0.003		1067.002	376.366	401583.528	6.6337694	-0.0015235	8.117x 10 ⁻⁰⁶	M= 1/(aL*bL)	1.6361044	
							0.5359425	-0.001525	Z=(1/bL)	-39.5320	
		Sum	1070.194	750.760	402429.502	15.099	-24.162	aL	bL	-0.025	

Table B 3: Langmuir B

Qe	Ce		Y =Ce/qe	X = Ce	XY	X^2	Matrix		Variable		Y
0.419	0.003		0.0068	0.0029	0.0000	0.0000	1.773	4	M= 1/bL		3.383
1.992	0.040		0.0203	0.0404	0.0008	0.0016	2.984	1.773	Z=(1/aL*bL)		0.909
3.313	1.727		0.5213	1.7270	0.9002	2.9827	Inverse				
0.001	0.003		2.8350	0.0027	0.0075	0.0000	-0.2016125	0.4548596	M= 1/bL	-0.2688411	
			0.0000	0.0000	0.0000	0.0000	0.3393629	-0.2016125	Z=(1/aL*bL)	0.9650109	
			0.0000	0.0000	0.0000	0.0000					
1.431		Sum	3.383	1.773	0.909	2.984	-3.720	bL	aL	-0.279	

Table B 4: Langmuir C

Qe	Ce		Y =qe	X = qe/Ce	XY	X^2	Matrix		Variable		Y
0.419	0.003		0.4189	146.2057	61.2388	21376.1008	197.783	4	M= 1/aL		5.725
1.992	0.040		1.9918	49.3058	98.2094	2431.0647	23810.970	197.783	Z=bL		165.805
3.313	1.727		3.3132	1.9184	6.3560	3.6803	Inverse				
0.001	0.003		0.0009	0.3527	0.0003	0.1244	-0.0035239	7.127E-05	M= 1/aL	-0.0083571	
			0.0000		0.0000	0.0000	0.4242421	-0.0035239	Z=bL	1.8444266	
1.431		Sum	5.725	197.783	165.805	23810.970	-119.659	aL	bL	1.844	

Table B 5: Langmuir E

Qe	Ce		Y =1/Ce	X = 1/qe	XY	X^2	Matrix		Variable		Y
0.419	0.003		349.0611	2.3875	833.3717	5.700	1070.194	-4	M=aL*bL		750.760
1.992	0.040		24.7539	0.5020	12.4276	0.252	1138500.160	-1070.194	Z=aL		402429.502
3.313	1.727		0.5790	0.3018	0.1748	0.091	Inverse				
0.001	0.003		376.3661	1067.0024	401583.5282	1138494.117	-0.00031396	1.173E-06	M=aL*bL	0.2365308	
							-0.33399972	0.000314	Z=aL	-124.4066	
1.431		Sum	750.760	1070.194	402429.502	1138500.160	-124.407	aL	bL	-0.002	

Table B 6: Langmuir D

Qe	Ce		Y =qe/Ce	X = qe	XY	X^2	Matrix		Variable		Y
0.419	0.003		146.2057	0.4189	61.2388	0.1754	-5.725	4	M=al		197.783
1.992	0.040		49.3058	1.9918	98.2094	3.9674	-15.120	5.725	Z=bL*aL		165.805
3.313	1.727		1.9184	3.3132	6.3560	10.9772	Inverse				
0.001	0.003		0.3527	0.0009	0.0003	0.0000	0.206622147	-0.1443695	M=al	16.92915	
							0.545718353	-0.2066221	Z=bL*aL	73.674723	
1.431		Sum	197.783	5.725	165.805	15.120	16.929	aL	bL	4.352	