

An Enhanced Application of Microsoft Excel Matrix Functions Technique for Adsorption Kinetics Models

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Abstract— Adsorption kinetics and equilibrium models are generally utilised to analyse the mechanism of adsorption characteristics in a set system and establish the mass-transfer saturation mechanisms. Previous studies were mainly focused on the description and utilization of the adsorption equilibrium models, evaluation of the appropriate quality, estimation of the model's parameters and practical application of adsorption process in the environmental pollution control such as water and wastewater treatment. However, insights into utilization of Microsoft Excel Matrix Functions Technique for Adsorption kinetics Models and their mathematical relations were rarely revealed in the literature. This paper provided step by step techniques required in the utilization of Microsoft Excel Matrix Functions in determination of adsorption kinetics parameters. Adsorption kinetics data were obtained from our previous and published articles. Unknown engineering design parameters in the commonly utilized adsorption kinetics models were determined using matrix functions and Microsoft Excel Solver in Microsoft Excel. Efficacies of these techniques were evaluated statistically using Analysis of Variance, Coefficient of Determination (CD) and correlation coefficient (R). and compared with experimental adsorption kinetics. The study revealed that adsorption treatment time and initial concentration of the pollutants are significant factors that influence adsorption kinetics at a 95 % confidence level ($F_{11,33} = 2.478$; $p = 0.022$ and $F_{3,33} = 65.997$, $p = 4.96 \times 10^{-14}$, which less than 0.05). It was concluded that the CD and R between experimental adsorption kinetics and simulated adsorption kinetics were 0.7692 and 0.8770, and 0.7975 and 0.8930 for Microsoft Excel solver and matrix techniques, respectively and Microsoft Excel are essential tools in the determination of engineering parameters in adsorption kinetics.

Keywords— *adsorption kinetics, Microsoft Excel Matrix Functions Technique, Microsoft Excel Solver, correlation coefficient*

I. INTRODUCTION

Environmental pollution is referred to the disproportionate presence of compounds, elements, and substances such as lead, mercury, cadmium, and chromium water, land, and the atmosphere [1]. These compounds, elements and substances may exist in nature, and due to human activities, such as smelting, mining, industrial production activities, volcanic eruptions, and weathering [2–25]. These compounds, elements and substances are also known as pollutants and are a times released into the environment [26–38]. Some of these pollutants are very harmful and toxic to human health, animals, and aquatic life; heavy metal and pollutants poisoning can originate

neurological diseases, malformations of human organ, cancers, and mutations [39–67], insomnia, children's mental disabilities, liver diseases, decreased immunity, visual disorders, depression, and skin diseases [68–87]. The presence of these pollutants can even root death in severe cases [88–107].

Environmental pollution not only harms flora and fauna, and disrupts the balance of ecosystems, it leads to a decrease in biodiversity and poses a grave safety danger to agricultural crops and soils.. Therefore, it is essential to provide effective and accurate measures to reduce or remove the effects and impacts of these stated pollutants [52–79]. There are several remediation techniques and methods for environmental pollution specifically water and wastewater treatment techniques [32–66]. The remediation techniques and methods include physical remediation methods, chemical remediation (precipitation, adsorption, chemical oxidation, chemical reduction, electrochemical treatment techniques and electrooxidation treatment techniques, membrane, and irradiation techniques) [23–88].

Fig. 1 presents some of the treatment techniques for the removal of these pollutants from the environment. Out of all these remediation techniques and methods, adsorption treatment has been found to be among the best [1–107]. Adsorption treatment process is one of the utmost used technologies in water and waste water treatment, based on its many advantages (easy maintenance, simple design, low price, and high efficiency [8–41]). Adsorption process is a typical mass transfer process in which the pollutants (conventional and emerging) from the liquid phase move to the solid adsorbent [29–35]. The adsorption equilibrium and kinetics models provide information and data on the adsorption rate, the mass transfer mechanisms and the performance of the adsorbent used, which indicates that accurate adsorption kinetics and equilibrium parameters are essential tools for the design of the adsorption treatment systems. Fig. 2 presents the adsorption mechanism of cationic pollutants by selected adsorbents

II. LITERATURE REVIEW

Analysis of the adsorption kinetics data can be conducted through linearized, non-linear, and least squares analysis using software such as Sigma Plot, Origin, Microsoft Excel Solver, MATLAB, Mathematica, Mathcad, Spreadsheet, Maple, GAUSS, Freemate, Flepro, Dataplot, Ardnial and SPSS.

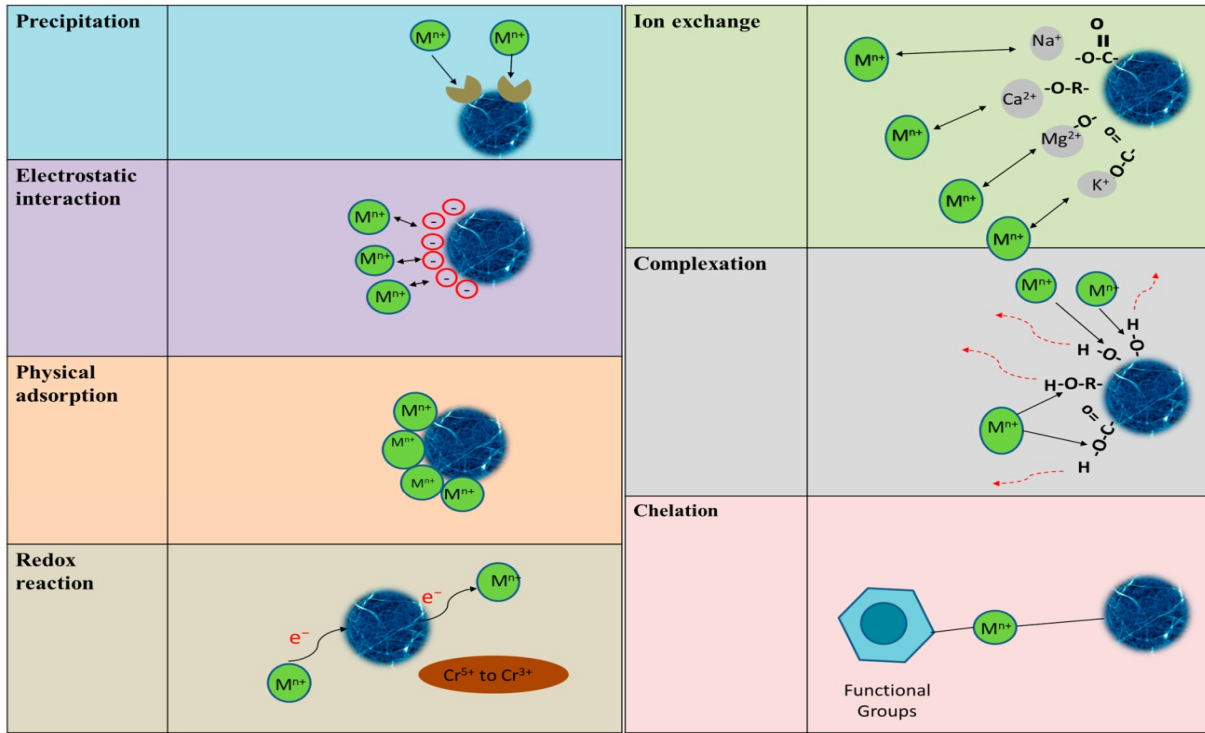


Fig. 2: Adsorption mechanism of pollutants by selected adsorbents (Source: Babajide et al.[42])

With reference to the cost of some of these software, there is a need to utilize tools and software that can be used to analyse data at no additional cost such as calc spread sheet on Libre Office, Microsoft Add-ins and Microsoft Excel. It has been documented that the functionality of Microsoft Excel

can significantly increase the robustness of numerical data analysis [42-46], but literature on utilization of matrix functions in Microsoft Excel for adsorption kinetics is rare and limited, hence this study.

III. METHODOLOGY

Data on arsenic removal from surface water such as Aponmu River, Ilara-Mokin, Nigeria by adsorption process using powdered eggshells and the adsorption properties of powdered eggshells were extracted from previous studies and published articles such as Amoko et al.[50]; Amoko et al. [51]; Fehintola et al.[52]; Obijole et al. [53] and Oke et al.[54]. Parameters in the adsorption kinetics models of Pseudo First and Second -Orders, Elovich, intraparticle diffusion and Chrastil were determined and analysed using matrix functions and Microsoft Excel Solver in Microsoft Excel. The adsorption kinetics models of Pseudo First and Second -Orders, Elovich, intraparticle diffusion and Chrastil are as follows:

$$\text{Pseudo -first - Order } q_t = q_e (1 - \exp^{-k_1 t}) \quad (1)$$

q_t means the solid phase concentration of sorbate (mg/mg) at time t , q_e means the equilibrium solid phase concentration of sorbate (mg/mg), t means the time, and k_1 means the rate constant of first-order pseudo adsorption

Pseudo -second - Order

$$q_t = \frac{q_e^2 k_2 t}{(1 + q_e k_2 t)} \quad (2)$$

q_e means the equilibrium solid phase concentration of sorbate (mg/mg), t means the time, k_2 means the rate constant of second-order pseudo adsorption.

Elovich equation

$$q_t = \frac{1}{\beta} \log_e(\alpha\beta) + \frac{1}{\beta} \log_e(t) \quad (3)$$

Where: β means the desorption constant during any experiment and α means the initial adsorption rate

Intraparticle diffusion

$$R = K_{id} t^a \text{ or } q_t = K_{id} t^{0.5} + C \quad (4)$$

R means the percentage of the pollutant adsorbed (%), 'a' means the gradient of linear plots and K_{id} means the intraparticle diffusion rate constant (/h).

$$\text{Chrastil } q_t = q_e (1 - \exp^{-K_c A_0 t})^n \quad (5)$$

where K_c means a rate constant ($\text{dm}^3/\text{g} \cdot \text{min}$), which depends on diffusion coefficients and the sorption capacity of biosorbent, A_0 means the dose of biosorbent (g/dm^3), and n means a heterogeneous structural diffusion resistance constant, which can range from 0 to 1. The results were compared with experimental adsorption kinetics. These results were evaluated statistically using ANOVA, CD and R. Fig. 3 and 4 present the flowchart of the procedures for

the matrix and the Microsoft Excel Solver techniques respectively. The mathematical and the matrix transformation adsorption kinetics models using least square expressions are as presented in equations (6) to (9). The details of the transformation are as presented in the appendix.

$$Y = mX + C \quad (6)$$

$$\sum_{i=1}^n Y_i = m \sum_{i=1}^n X_i + nC \quad (7)$$

IV. RESULTS AND DISCUSSION

Results from the study are presented as follows: Step-by step of Microsoft Excel Matrix Functions techniques, Adsorption equilibrium model and parameters, efficacies evaluations of the technique.

a) *Step-by step of Microsoft Excel Matrix Functions techniques for adsorption kinetics determination:* Fig. 5 and 6 provide data, information and deep insight into procedures needed to utilize Microsoft Excel Matrix functions and Microsoft Excel Solver to solve adsorption kinetics equations. These figures provide the actions and solutions to the flow chart provided in Fig. 3 and 4.

b) *Adsorption Kinetics models and their parameters:* Table 1 presents the adsorption kinetics models and the values of adsorption capacities obtained using the two techniques (matrix and Microsoft Excel Solver). The Table established that the values of adsorption kinetics parameters vary with the adsorption kinetics models, methods and techniques used (matrix and Microsoft Excel Solver), but the values were similar in magnitudes. This result indicates that the matrix technique in the Microsoft Excel (in-built functions) can be used to solve numerical problems of adsorption kinetics models. The Table also established that these adsorption kinetics models must be linearized and transformed into matrix arrays for easier applications. The detail computation and linearization are presented at the appendices.

c) *Statistical Evaluations of Microsoft Matrix Functions in Microsoft Excel:* Table 2 presents the results of the R for both matrix and Microsoft Excel Solver

$$\sum_{i=1}^n X_i Y_i = m \sum_{i=1}^n X_i^2 + C \sum_{i=1}^n X_i \quad (8)$$

Matrix transformation of

$$\begin{pmatrix} \sum_{i=1}^n X_i & n \\ \sum_{i=1}^n X_i^2 & \sum_{i=1}^n X_i \end{pmatrix} \begin{pmatrix} m \\ C \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n Y_i \\ \sum_{i=1}^n X_i Y_i \end{pmatrix} \quad (9)$$

techniques. These values of R established that the relationship between experimental adsorption equilibria and simulated adsorption equilibria was between 0.8599 (Pseudo First order kinetics) and 0.9882 (Pseudo Second order kinetics), which indicates that the relationship was between good to very good relationship (Loveday 1980; Oke, 2008; Oke et al., 2018). These results indicate the matrix technique is a vital tool for solving numerical problems such as adsorption kinetics. A low value of Pseudo First order kinetics indicates that equations such as Pseudo First order kinetics should be checked or minized. Pseudo -first – Order equation can be expressed as

$$q_t = q_e (1 - \exp^{-k_1 t});$$

linear as

$$\ln(q_e - q_t) = \ln q_e - k_1 t$$

or as

$$\log(q_e - q_t) = \log q_e - \frac{k_1}{2.303} t$$

A high value of Pseudo Second order kinetics indicates that equations such as Pseudo Second order kinetics should be encouraged. The utilization of the matrix technique for Chrastil requires further steps and procedures. Fig. 7 revealed that the CD and R between experimental adsorption kinetics and simulated adsorption kinetics were 0.7692 and 0.8770, and 0.7975 and 0.8930 for Microsoft Excel solver and matrix techniques, respectively. This result indicates the performance of the systems were similar and there was no significant difference between the two techniques at 95 % confidence level ($F_{1,2} = 0.0945$ and $p = 0.7876$).

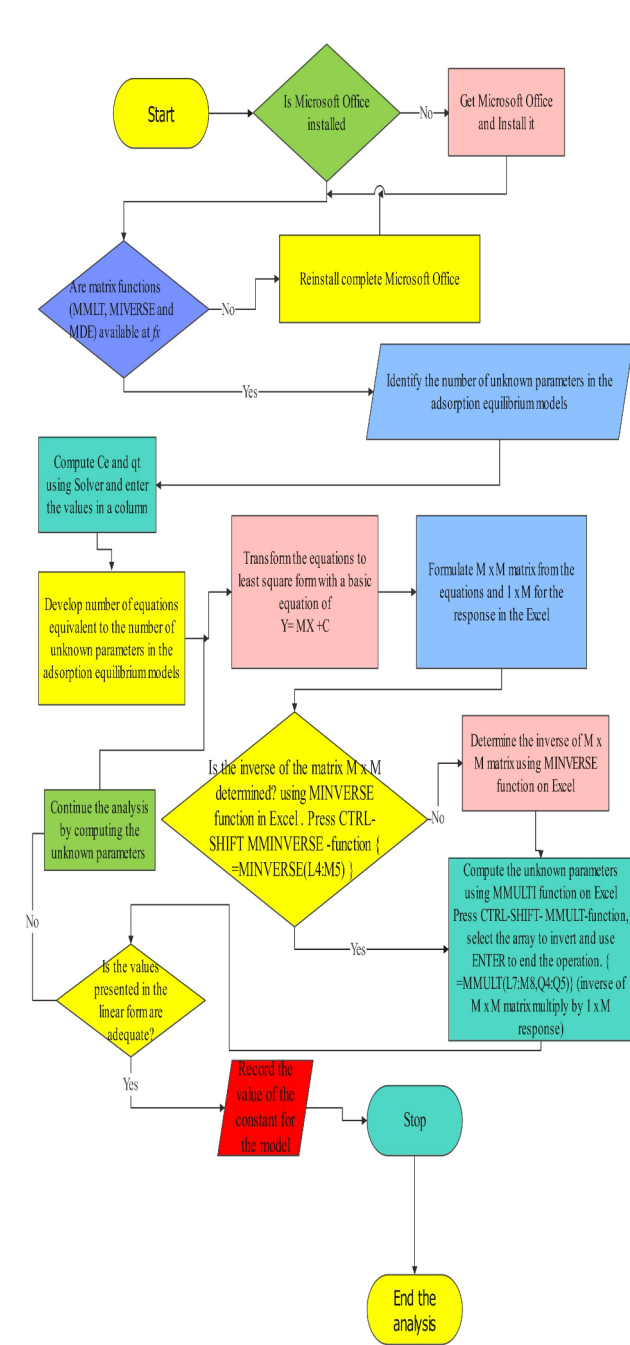


Fig. 3: Procedure for using the Matrix technique on Microsoft Excel in the computation of the adsorption kinetics' parameters

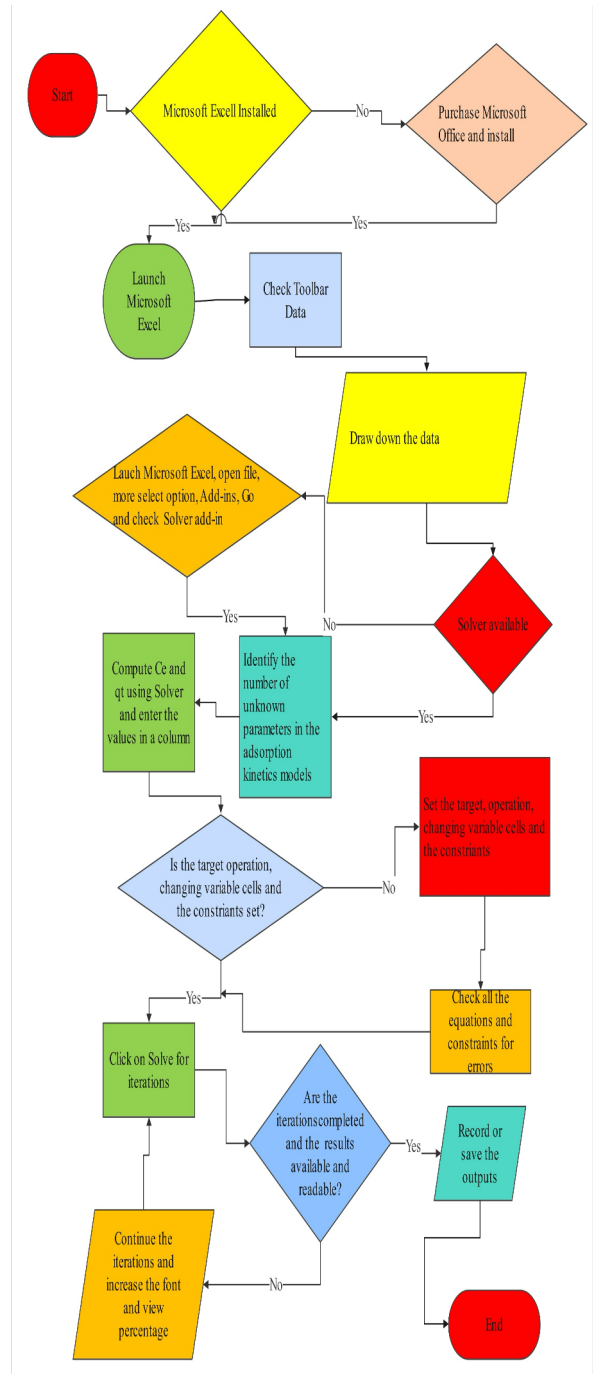


Fig. 4: Procedure for using Microsoft Excel Solver technique on Microsoft Excel in the computation of the Isotherm's parameters

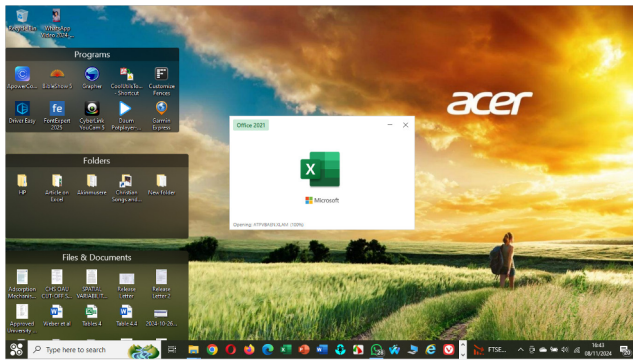


Fig. 5ai: Launching of Microsoft Excel on the desktop

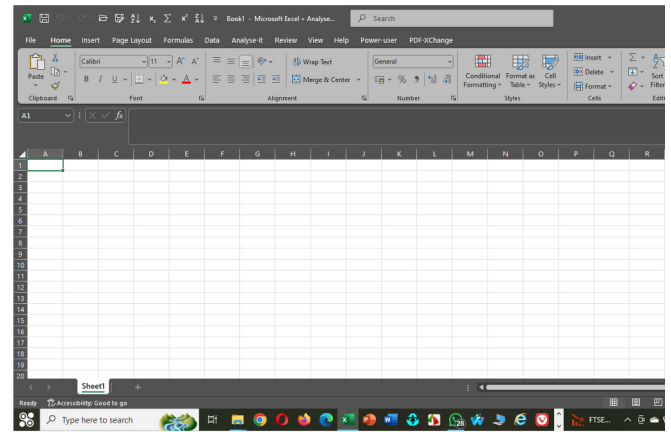


Fig. 5aii: Fully Launched Microsoft Excel

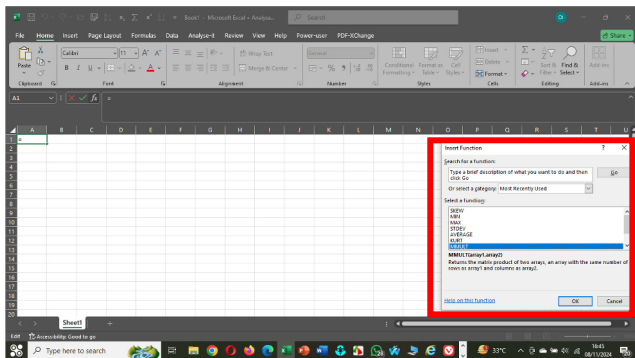


Fig. 5aiii: Fully Launched Microsoft with fx dialogue

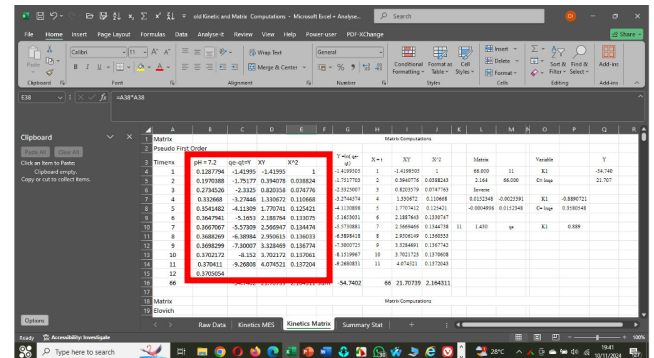


Fig. 5b: Fully launched Microsoft Excel with Ct and qt entered

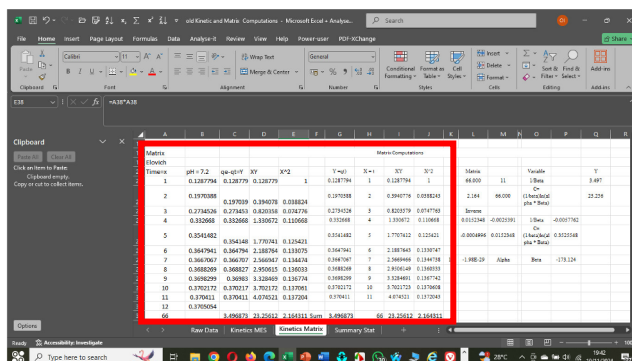


Fig. 5c: Fully launched Microsoft Excel with least square computations

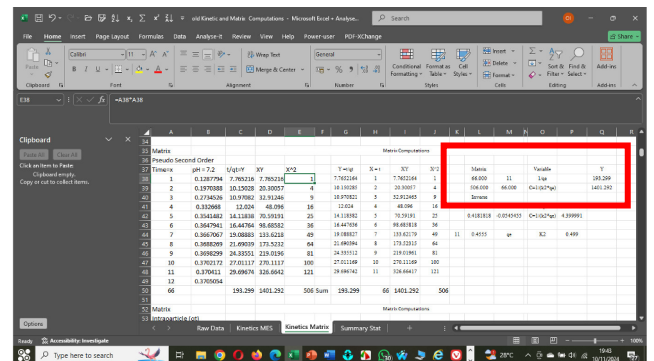


Fig. 5d: Fully launched Microsoft Excel with Matrix array entered

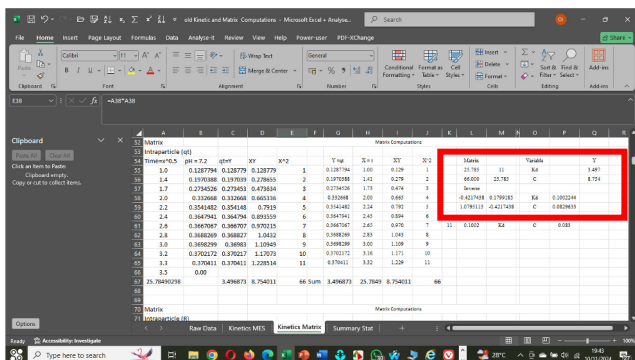


Fig. 5e: Fully launched Microsoft Excel with Matrix functions

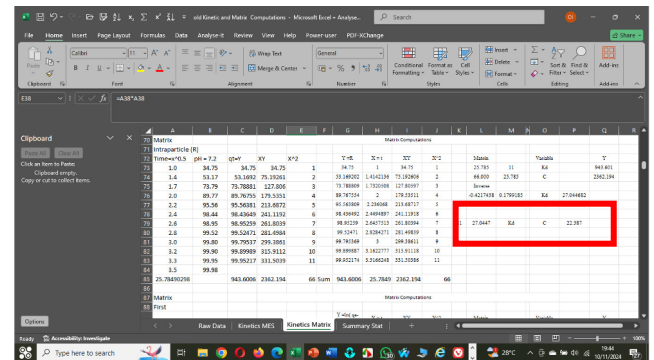


Fig. 5f: Fully launched Microsoft Excel with the final answers

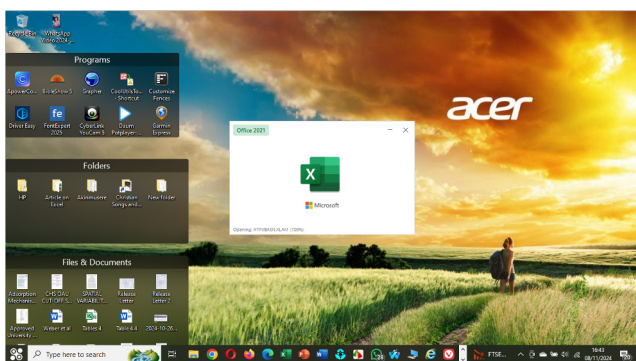


Fig. 6ai: Launching of Microsoft Excel on the desktop for MES

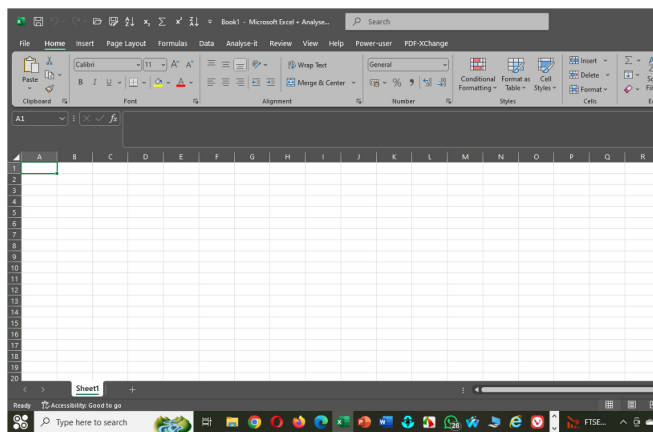


Fig. 6aii: Fully Launched Microsoft Excel for MES

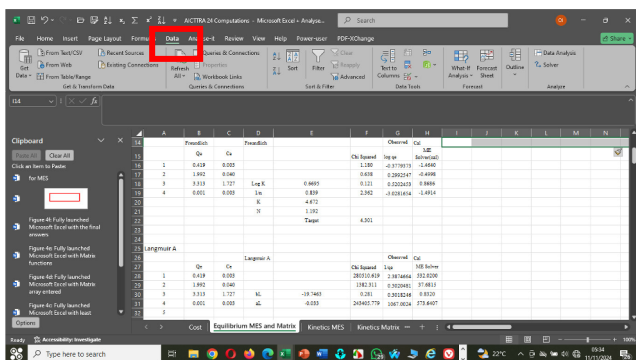


Fig. 6aiii: Fully Launched Microsoft with Data on toolbar MES dialogue

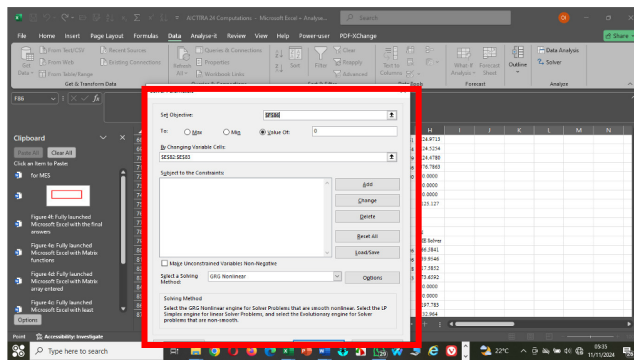


Fig. 6aiv: Fully Launched Microsoft with MES dialogue

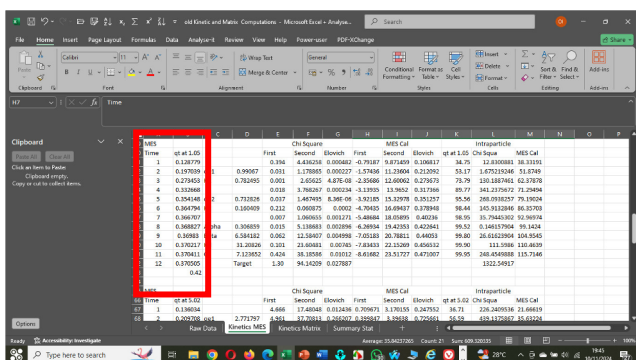


Fig. 6b: Fully launched Microsoft Excel with Ct and qt entered

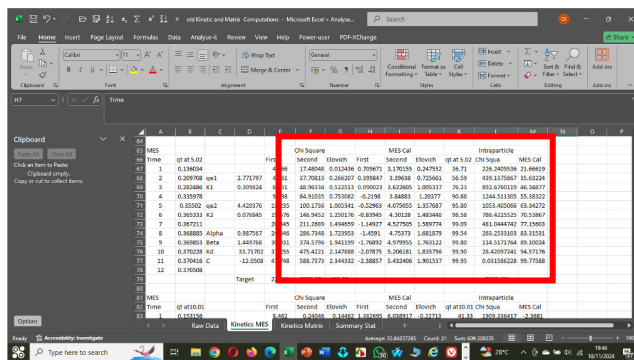


Fig. 4b: Fully launched Microsoft Excel with MES settings

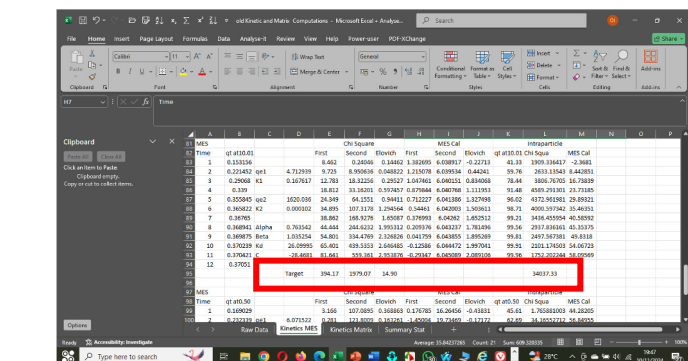


Fig. 6d: Fully launched Microsoft Excel with MES targets

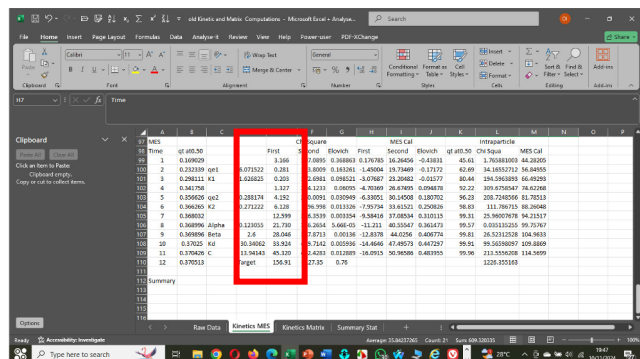


Fig. 6f: Fully launched Microsoft Excel with MES the final answers

Table 1: the adsorption kinetics models and the values of adsorption capacities obtained using the two techniques

Summary		MES	Matrix
Intraparticle diffusion	K_d	0.2070	0.1410
$q_t = K_{id}t^{0.5} + C$	C	0.4851	0.0180
Pseudo -first - Order	q_e	0.9907	1.5094
$q_t = q_e (1 - \exp^{-k_1 t})$; $\ln(q_e - q_t) = \ln q_e - k_1 t$ or $\log(q_e - q_t) = \log q_e - \frac{k_1}{2.303} t$	K_1	0.7825	0.8528
Pseudo -second – Order	q_e	0.7328	0.7328
$q_t = \frac{q_e^2 k_2 t}{(1 + q_e k_2 t)}$; $\frac{t}{q_t} = \frac{t}{q_e} + \frac{1}{k_2 q_e^2}$ or $\frac{1}{q_t} = \frac{(1 + q_e k_2 t)}{q_e^2 k_2 t} = \frac{1}{q_e^2 k_2 t} + \frac{1}{q_e}$	K_2	0.1604	0.1604
Elovich	Alpha	0.3069	0.0000
$q_t = \frac{1}{q_e} \ln(\alpha \beta t + 1)$ or $q_t = \frac{\ln(\alpha \beta)}{\beta} + \frac{\ln(t)}{\beta}$	Beta	6.5842	-29.6762
Intraparticle diffusion	K_d	31.2083	33.5675
$R = K_{id}t^a$ or $\ln(R) = \ln(K_{id}) + a \ln(t)$	C	7.1237	4.2851
	q_e	0.4206	
Chrastil	A	0.9174	
$q_t = q_e (1 - \exp^{-K_c A_0 t})^n$	K	0.9174	
	n	3.6960	

Table 2: the results of the R for both matrix and Microsoft Excel Solver techniques

Statistical Evaluation	Intraparticle diffusion	Pseudo First-order Kinetics	Pseudo Second-order Kinetics	Elovich
Coefficient of Determination of Matrix Technique	0.8764	0.7579	0.9648	0.7984
Coefficient of Determination of Microsoft Excel Solver Technique	0.9362	0.8706	0.9822	0.8935
Correlation of Coefficient of Matrix Technique	0.8784	0.7464	0.9766	0.7896
Correlation of Coefficient of Microsoft Excel Solver Technique	0.9372	0.8639	0.9882	0.8886

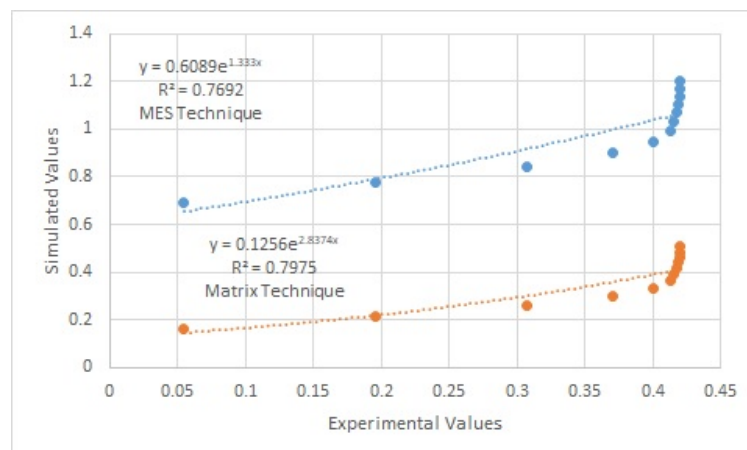


Fig 7: Relationship between experimental values and simulated values from the two techniques

V. CONCLUSION

Based on the study and the findings, the following key conclusions can be drawn:

- Matrix technique with the Microsoft Excel function tools can be used to solve adsorption kinetics equations;
- Using the matrix is flexible and faster due to its capacities as in-built functions,
- The performance of the evaluated adsorption kinetics models was in the order of Pseudo-second-order greater than intraparticle diffusion greater than Elovich greater than pseudo-first-order kinetic

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APPENDICES

DERIVATIONS OF ADSORPTION KINETICS EQUATIONS AND GENERAL FORM OF LEAST SQUARE FOR MATRIX TRANSFORMATIONS

APPENDIX A: Derivations Of Adsorption Kinetics Equations

A. Pseudo Second order Model

$$q_t = \frac{K_2 q_e^2 t}{1 + q_e K_2 t} \quad \text{which indicates that}$$

$$\frac{1}{q_t} = \frac{1 + q_e K_2 t}{q_e^2 K_2 t} = \frac{1}{q_e^2 K_2 t} + \frac{q_e K_2 t}{q_e^2 K_2 t} = \frac{1}{q_e^2 K_2 t} + \frac{1}{q_e}$$

$$\frac{t}{q_t} = \frac{1}{q_e^2 K_2} + \frac{t}{q_e} = \left(\frac{1}{q_e^2} \right) \left(\frac{1}{K_2} \right) + \left(\frac{1}{q_e} \right) t \quad \text{transform}$$

$$\text{to } \frac{t}{q_t} = \frac{1}{K_2} \left(\frac{1}{q_e} \right)^2 + \left(\frac{1}{q_e} \right) t \quad \text{which is equivalent to}$$

$Y = mX + C$ where X stand for independent variables = t and Y stands for dependent variables

B. Pseudo First Order Model

$$q_t = q_e (1 - \text{Exp}^{-K_1 t}) = q_e - q_e \text{Exp}^{-K_1 t} \quad \text{which}$$

indicates that $q_t - q_e = -q_e \text{Exp}^{-K_1 t}$ and

$$-q_t + q_e = q_e \text{Exp}^{-K_1 t} = q_e - q_t$$

$$\ln(q_e - q_t) = -K_1 t + \ln(q_e) \quad \text{which is equivalent to}$$

$Y = -mX + C$ where X stand for independent variables = t and Y stands for dependent variables

C. Elovich equation

$$q_t = \frac{1}{\beta} \log_e(\alpha\beta) + \frac{1}{\beta} \log_e(t) \quad \text{which is}$$

equivalent to $Y = mX + C$ where X stand for independent variables = t and Y stands for dependent variables

D. Intraparticle diffusion:

$$R = K_{id} t^a \text{ or } q_t = K_{id} t^{0.5} + C$$

a. $R = K_{id} t^a$ which can be transformed to

$$\ln(R) = \ln(K_{id}) + (a) \ln(t) \quad \text{which is}$$

equivalent to $Y = C + mX$ where X stand for independent variables = t and Y stands for dependent variables

b. $q_t = K_{id} t^{0.5} + C$ which is equivalent to

$Y = mX + C$ where X stand for independent variables = t and Y stands for dependent variables

APPENDIX B: General Form of Least Square For Matrix Transformations

Pseudo First Order Model

Time=x	pH = 7.2	qe-qt=Y	XY	X^2	Y =ln(qe-qt)	X = t	XY	X^2	Matrix		Variable	Y	
1	0.1287794	-1.41995	-1.41995	1	-1.419950522	1	-1.419950522	1	66.000	11	K1	-54.740	
2	0.1970388	-1.75177	0.394078	0.038824	-1.75177031	2	0.394077613	0.038824291	2.164	66.000	C= lnqe	21.707	
3	0.2734526	-2.3325	0.820358	0.074776	-2.332500719	3	0.82035794	0.07477635	Inverse				
4	0.332668	-3.27446	1.330672	0.110668	-3.274457446	4	1.330671983	0.110667995	0.01523478	-0.00253913	K1	-0.889072068	
5	0.3541482	-4.11309	1.770741	0.125421	-4.113089817	5	1.770741163	0.125420971	-0.000499588	0.01523478	C= lnqe	0.358054765	
6	0.3647941	-5.1653	2.188764	0.133075	-5.165303083	6	2.188764341	0.133074704					
7	0.3667067	-5.57309	2.566947	0.134474	-5.573088126	7	2.566946599	0.134473772	11	1.430	qe	K1	0.889
8	0.3688269	-6.38984	2.950615	0.136033	-6.389841822	8	2.950614934	0.136033258					
9	0.3698299	-7.30007	3.328469	0.136774	-7.300072497	9	3.328469087	0.136774154					
10	0.3702172	-8.152	3.702172	0.137061	-8.151996662	10	3.702172289	0.137060797					
11	0.370411	-9.26808	4.074521	0.137204	-9.26808308	11	4.07452099	0.137204308					
12	0.3705054												
66		-54.7402	21.70739	2.164311	Sum	-54.7402	66	21.70739	2.164311				

Elovich equation

Time=x	pH = 7.2	qe-qt=Y	XY	X^2	Y =qt	X = t	XY	X^2	Matrix	Variable
1	0.1287794	0.128779	0.128779	1	0.128779412	1	0.128779412	1	66.000	11 1/Beta
2	0.1970388	0.197039	0.394078	0.038824	0.197038807	2	0.394077613	0.038824291	2.164	66.000 C= (1/beta)ln(alpha a * Beta)
3	0.2734526	0.273453	0.820358	0.074776	0.273452647	3	0.82035794	0.07477635	Inverse	
4	0.332668	0.332668	1.330672	0.110668	0.332667996	4	1.330671983	0.110667995	0.01523478	-0.00253913 1/Beta C=
5	0.3541482	0.354148	1.770741	0.125421	0.354148233	5	1.770741163	0.125420971	-0.000499588	0.01523478 (1/beta)ln(alpha a * Beta)
6	0.3647941	0.364794	2.188764	0.133075	0.364794057	6	2.188764341	0.133074704		
7	0.3667067	0.366707	2.566947	0.134474	0.366706657	7	2.566946599	0.134473772	11	-1.98E-29 Alpha Beta
8	0.3688269	0.368827	2.950615	0.136033	0.368826867	8	2.950614934	0.136033258		
9	0.3698299	0.36983	3.328469	0.136774	0.369829899	9	3.328469087	0.136774154		
10	0.3702172	0.370217	3.702172	0.137061	0.370217229	10	3.702172289	0.137060797		
11	0.370411	0.370411	4.074521	0.137204	0.370410999	11	4.07452099	0.137204308		
12	0.3705054									
66		3.496873	23.25612	2.164311	Sum	3.496873	66	23.25612	2.164311	

Pseudo Second order Model

Time=x	pH = 7.2	t/qt=Y	XY	X^2	Y =t/qt	X = t	XY	X^2	Matrix	Variable	Y		
1	0.1287794	7.765216	7.765216	1	7.765216398	1	7.765216398	1	66.000	11	1/qe	193.299	
2	0.1970388	10.15028	20.30057	4	10.15028478	2	20.30056957	4	506.000	66.000	C=1/(k2*qe)	1401.297	
3	0.2734526	10.97082	32.91246	9	10.97082086	3	32.91246258	9	Inverse				
4	0.332668	12.024	48.096	16	12.02400005	4	48.09600021	16	-0.054545455	0.009090909	1/qe	2.195440654	
5	0.3541482	14.11838	70.59191	25	14.11838191	5	70.59190954	25	0.418181818	-0.054545455	C=1/(k2*qe)	4.399991038	
6	0.3647941	16.44764	98.68582	36	16.44763638	6	98.68581828	36					
7	0.3667067	19.08883	133.6218	49	19.08882717	7	133.6217902	49	11	0.4555	qe	K2	0.499
8	0.3688269	21.69039	173.5232	64	21.69039385	8	173.5231508	64					
9	0.3698299	24.33551	219.0196	81	24.33551218	9	219.0196096	81					
10	0.3702172	27.01117	270.1117	100	27.01116863	10	270.1116863	100					
11	0.370411	29.69674	326.6642	121	29.69674234	11	326.6641657	121					
12	0.3705054												
66		193.299	1401.292	506	Sum	193.299	66	1401.292	506				

Intraparticle diffusion:

Intraparticle (qt)													
Time=x^0.5	pH = 7.2	qt=Y	XY	X^2	Y =qt	X = t	XY	X^2	Matrix	Variable	Y		
1.0	0.1287794	0.128779	0.128779	1	0.128779412	1.00	0.129	1	25.785	11	Kd	3.497	
1.4	0.1970388	0.197039	0.278655	2	0.197038807	1.41	0.279	2	66.000	25.785	C	8.754	
1.7	0.2734526	0.273453	0.473634	3	0.273452647	1.73	0.474	3	Inverse				
2.0	0.332668	0.332668	0.665336	4	0.332667996	2.00	0.665	4	-0.421743836	0.179918544	Kd	0.100224409	
2.2	0.3541482	0.354148	0.7919	5	0.354148233	2.24	0.792	5	1.079511263	-0.421743836	C	0.082963284	
2.4	0.3647941	0.364794	0.893559	6	0.364794057	2.45	0.894	6					
2.6	0.3667067	0.366707	0.970215	7	0.366706657	2.65	0.970	7	11	0.1002	Kd	C	0.083
2.8	0.3688269	0.368827	1.0432	8	0.368826867	2.83	1.043	8					
3.0	0.3698299	0.36983	1.10949	9	0.369829899	3.00	1.109	9					
3.2	0.3702172	0.370217	1.17073	10	0.370217229	3.16	1.171	10					
3.3	0.370411	0.370411	1.228514	11	0.370410999	3.32	1.229	11					
3.5	0.00												
25.78490298		3.496873	8.754011	66	Sum	3.496873	25.7849	8.754011	66				

Intraparticle (R)													
Time=x^0.5	pH = 7.2	qt=Y	XY	X^2		Y =R	X = t	XY	X^2	Matrix		Variable	Y
1.0	34.75	34.75	34.75	1		34.75	1	34.75	1	25.785	11	Kd	943.601
1.4	53.17	53.1692	75.19261	2		53.16920177	1.414213562	75.19260624	2	66.000	25.785	C	2362.194
1.7	73.79	73.78881	127.806	3		73.78880941	1.732050808	127.8059669	3	Inverse			
2.0	89.77	89.76755	179.5351	4		89.76755444	2	179.5351089	4	-0.421743836	0.179918544	Kd	27.04468191
2.2	95.56	95.56381	213.6872	5		95.5638088	2.236067977	213.6871727	5	1.079511263	-0.421743836	C	22.38691799
2.4	98.44	98.43649	241.1192	6		98.43649152	2.449489743	241.1191763	6				
2.6	98.95	98.95259	261.8039	7		98.95258999	2.645751311	261.8039447	7	11	27.0447	Kd	22.387
2.8	99.52	99.52471	281.4984	8		99.52471006	2.828427125	281.4983895	8				
3.0	99.80	99.79537	299.3861	9		99.79536946	3	299.3861084	9				
3.2	99.90	99.89989	315.9112	10		99.89988718	3.16227766	315.9111815	10				
3.3	99.95	99.95217	331.5039	11		99.95217436	3.31662479	331.5038593	11				
3.5	99.98												
25.78490298		943.6006	2362.194	66 Sum		943.6006	25.7849	2362.194	66				